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## Low Frequency Noise and Bubble Plume Oscillations

Presented at the  
114th Meeting of the  
Acoustical Society of America,  
16-20 November 1987, Miami, Florida

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### Preface

This report contains the oral presentation "Low Frequency Noise and Bubble Oscillations" given at the 114th Meeting of the Acoustical Society of America, 16-20 November 1987, in Miami, Florida.

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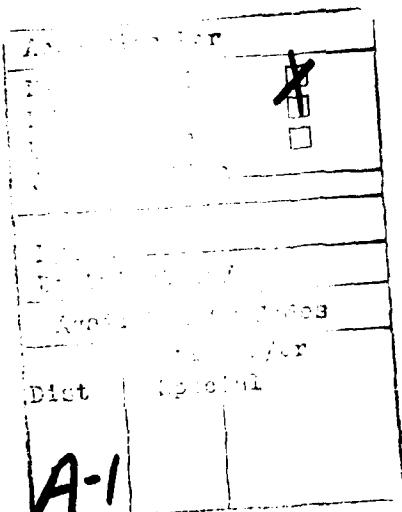
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of the sea surface, only the resulting dipole source would be of importance. These regions could also have a resonant characteristic, and when driven by the turbulence result in sufficient radiated sound. These results are similar to and consistent with the analysis by A. Prosperetti - "Natural Mechanisms of Surface Generated Noise in the Ocean", SEA SURFACE SOUND, B. Kerman (ed), Kluwer Acad. Pub., Boston, 151-171, 1988.

7/11/03. Keweenaw Upland Acoustics.  
3. Subcloud - 1000 ft. 5000 ft.  
Landscape - 1000 ft. 5000 ft.  
Elevation - 1000 ft. 5000 ft.

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## LOW-FREQUENCY OCEAN SURFACE NOISE SOURCES

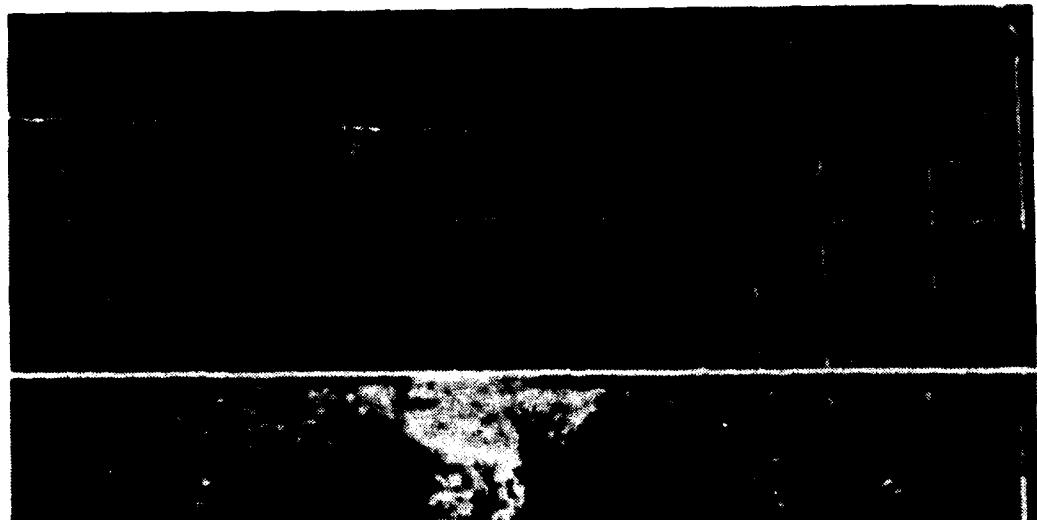
W. CAREY (J. ACOUSTIC SOC. AM. 78(SI), 1985)

### CONCLUSIONS:

- $f < 2$  Hz: WAVE-WAVE INTERACTION (KIBBLEWHITE).
- $20 < f < 200$  Hz: DATA (VERTICAL/OMNI) SUGGEST WIND NOISE IS IMPORTANT.
- $f < 200$  Hz VERTICAL DIR. DET. BY DSE AND SSC.
- $20 < f < 200$  Hz: THE SOURCE OF NOISE.
  - $BF < 5, SS < 3, WS < 8 - 10$  m/s:  
WAVE TURBULENCE INTERACTION (YEN & PERRONE, GONCHAROV)
  - $BF > 5, SS > 3, WS > 8 - 10$  m/s:  
BUBBLE NOISE-THAT COLLECTIVE BUBBLE OSCILLATIONS DRIVEN BY THE PSEUDO-SONIC AND TURBULENT PRESSURE FIELDS. DIPOLE DUE TO PRESSURE RELEASE SURFACE.
  - TURBULENCE SPECTRA AND VOID FRACTION DATA REQUIRED.

The evidence for a wind dependence in low-frequency ambient noise ( $< 200$  Hz) is sparse due to the corrupting influences of the radiated sound from ships. Measurements made with hydrophones below critical depth, in sparse shipped basins or at high sea states, show indications of two distant regimes associated with the occurrence of breaking waves and a variation of root-mean-square pressure with the square of the local wind speed. In a previous paper [W. Carey, J. Acoust. Soc. Am. Supl. 1 78, S1 (1985)], the generation of sound by wave turbulence interaction at low sea states ( $WS < 10$  m/s) and collective bubble oscillations driven by turbulence at high sea states was proposed ( $WS > 10$  m/s). This paper shows that recently observed bubble clouds that penetrate to tens of meters below the surface of the sea result in regions of low sonic velocity described by Wood's treatment of air-bubble water mixtures, but with a density close to that of water. It is shown that such a region can be treated as a flexible body with mixture speed and density. The radiation from such a body would be monopole and dipole in nature; but due to the proximity of the sea surface, only the resulting dipole source would be of importance. These regions could also have a resonant characteristic and, when driven by the turbulence, result in sufficient radiated sound. These results are similar to and consistent with the analysis by A. Prosperetti [Nato ASI, "Natural Mechanisms of Surface Generated Noise in the Ocean," (1987)].

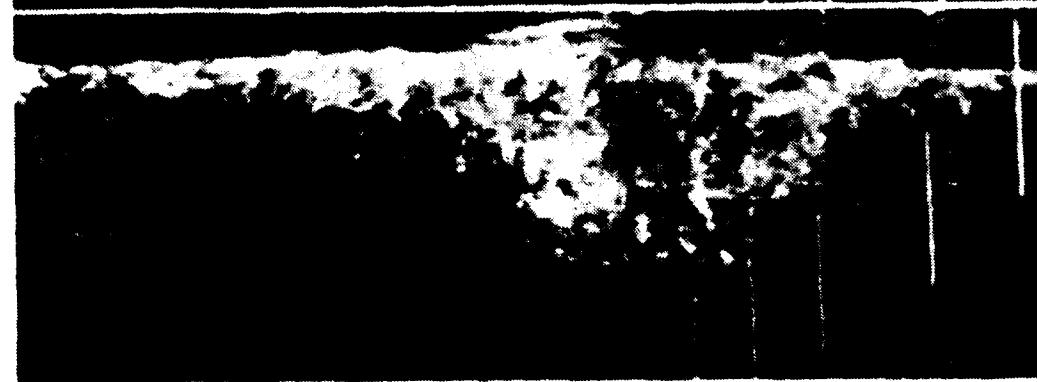
(a)



(b)



(c)



(d)



**BACKGROUND:**

This vu-graph shows the results of Su et al (1984) and illustrates the production of bubble clouds when wave breaking occurs in tank water. The picture is striking and is not what one would observe when wavebreaking occurs in salt water. The difference being the coalescence of micro-bubbles in fresh water to produce larger bubbles. The ionic properties of salt water (Pounder (1986)) prevent coalescence; consequently plumes of micro-bubbles have been observed to penetrate to depths on the order of tens of meters.

In a previous paper (Carey (1985)) a review of measured vertical and omnidirectional ambient noise was presented. Experimental evidence obtained with hydrophones in the Northern Hemisphere below critical depth in the deep ocean (Whittenborn (1976)) and in the Southern Hemisphere (Burgess (1983)), Browning (1982) showed the existence of wind produced noise in the 200Hz region. The data showed agreement with the observations of Kerman that at frequencies greater than 500 Hz the noise produced at the sea surface has a different speed dependent characteristic above and below a critical wind speed of 12 m/sec ( $U_c = 4gT/\rho$ ), thus implying two different physical mechanisms for the production of sound.

Examination of the possible ways the wind may produce sound (Carey (1985), (1987)) showed that wave-wave interactions would be important at ultra-low frequencies <2Hz, individual bubbles, spray and splashes at the higher frequencies >2kHz, and wave-turbulence interaction in the low to mid-frequencies 200Hz. However, wave turbulence interaction could not explain the levels observed at higher wind speeds or explain the rapid transition in levels observed in experimental data at the critical wind speed.

Thus a possible explanation of the observed effect was that for wind speeds greater than 8-10m/s, noise produced near the sea surface was due to the collective oscillations of micro-bubble clouds driven by the pseudo-sonic and turbulent pressure fields. If the bubble plume or portion thereof could be considered as a compact source of sound, then in the absence of the pressure release surface it could be represented as a sum of a monopole, dipole, quadrupole, plus higher order terms. The addition of the pressure release surface would result in the monopole term

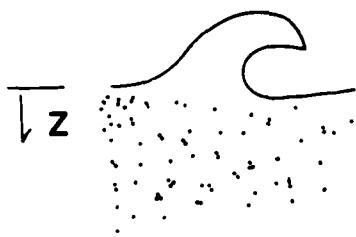
## BREAKING WAVES



SPILLING

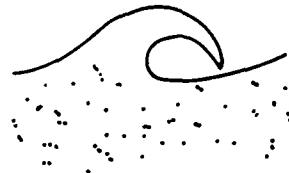


PLUNGING



RESIDUAL BUBBLES

$$N(Z) = N_o \exp(-\alpha Z)$$



BUBBLE PLUME

becoming a dipole-like term that would govern the radiation characteristic. These concepts are discussed farther in a recent paper (Carey (1986)) and are similar to the analytical work of Prosperetti (1985, 1986).

## **SUMMARY**

- THIS PAPER EXAMINES THE POSSIBILITY THAT BUBBLE PLUMES FROM BREAKING WAVES CAN PRODUCE MF/LF ( $f \sim 200$  Hz) SOUND.
- THE PROPOSED PHYSICAL MECHANISM IS BASED ON:
  - THE PROPERTIES OF BUBBLY MIXTURES
  - THE RESONANT OSCILLATION OF A BUBBLY VOLUME, CLOUD.
- WE SHOW THAT THE RANGE OF RESONANT FREQUENCIES SPANS THE REGION OF INTEREST AND THAT IF VOLUME FRACTIONS IN THE CLOUDS (PLUMES, AS OPPOSED TO THE RESIDUAL BUBBLE LAYER) ARE IN THE 0.1 TO 1% RANGE ( $VF = .001$  TO  $.01$ ), THEN LEVELS ON THE ORDER OF 60 dB RE  $1 \mu\text{Pa}$  CAN BE REALIZED.
- WE RECOMMEND THAT VOID FRACTION MEASUREMENTS BE PERFORMED.

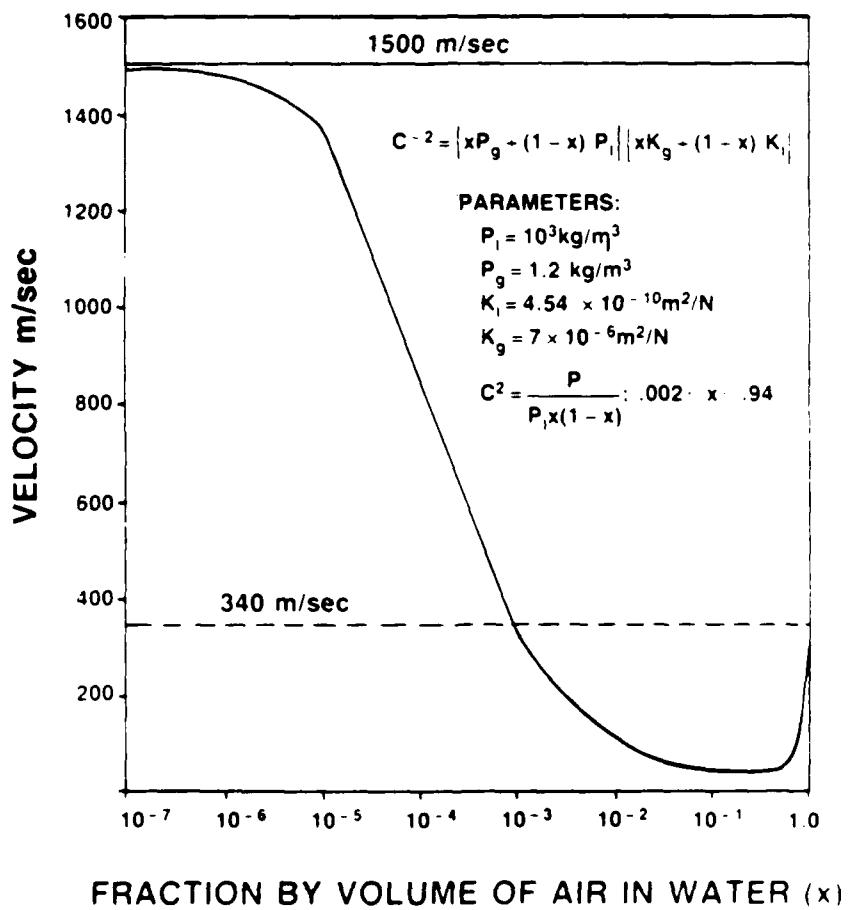
**SUMMARY:**

This paper examines the possibility that bubble plumes from breaking waves can produce sufficient sound in the low-frequency (LF) to mid-frequency (MF) region ( $f < 2000\text{Hz}$ ). As shown in this vu-graph, we are distinguishing between the residual bubble layer which decreases exponentially with depth and the plume of micro-bubbles which can be convected to depths on the order of ten meters below the breaking wave.

The proposed physical mechanism is based on the acoustic properties of a bubbly-liquid mixture and the resonant oscillation of a bubbly volume (cloud or plume). We show that the range of resonant frequencies spans the region of interest (20 Hz to 2 kHz) and that if the volume fraction of the bubbles in the cloud is between  $10^{-3}$  (.1%) to  $10^{-2}$  (1%) then sound levels on the order of 60dB re  $1\mu\text{Pa}$  can be realized.

We also conclude that void fraction measurements need to be performed beneath the breaking waves rather than bubble size and population measurements. Finally we observe that "a good radiator of sound is also a good scatterer of sound."

## LOW FREQUENCY APPROXIMATION: WOOD'S RESULTS



### The Low Frequency Approximation: Wood's Result

A. B. Wood (1932) showed that the sonic speed could be calculated for an air-bubble/water mixture by use of the mixture density ( $\rho_m$ ) and the mean compressibility ( $k_m$ ). The mixture can be treated as a continuous medium when the bubble diameter ( $d$ ) and spacing between the bubbles ( $D$ ) are much less than the wavelength of sound. In the case of low frequencies, for the mixture with a volume fraction ( $X$ ) of gas we can calculate the mean density and compressibility as follows:

$$\rho_m = (1-X)\rho_e + X\rho_g$$

$$k_m = (-1/V_m) \frac{dV_m/dP}{} = (-1/V_e) \left( \frac{dV_e/dP}{V_e/V_m} \right) + \left( \frac{1}{V_g} \right) \left( \frac{dV_g/dP}{V_g/V_m} \right) = (1-X)k_e + Xk_g$$

This implies that a state of equilibrium prevails and the mixture mass is conserved, and the pressure,  $P$ , is uniform throughout the mixture (a low frequency assumption). Since the sonic speed is

$$c^2 \equiv dP/d\rho = (\rho k)^{-1}$$

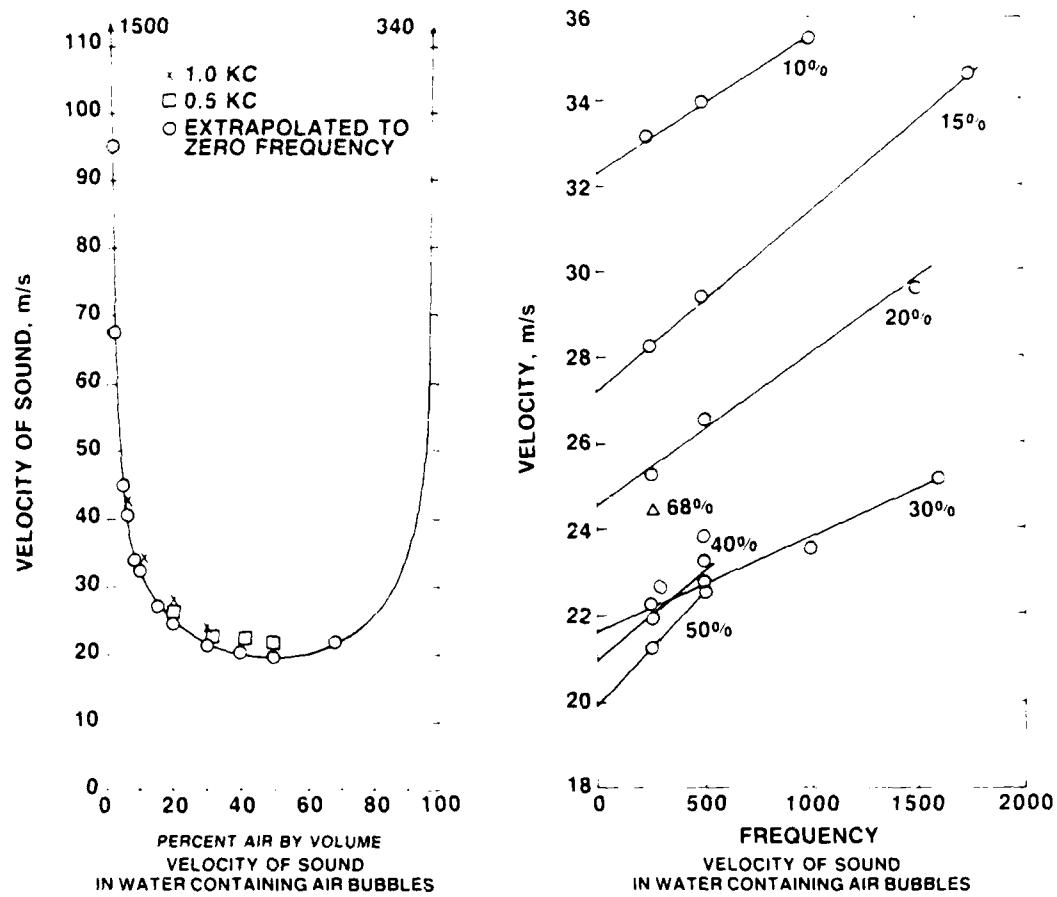
$$\text{we have } C_m^{-2} = C_{m,ef}^{-2} = [(1-X)\rho_e + X\rho_g][(1-X)k_e + Xk_g]$$

$$C_m^{-2} = (1-X)^2 C_e^{-2} + X^2 C_g^{-2} + (X)(1-X)(\rho_e C_g^2 + \rho_g^2 C_e^2) / (\rho_e \rho_g C_e^2 C_g^2)$$

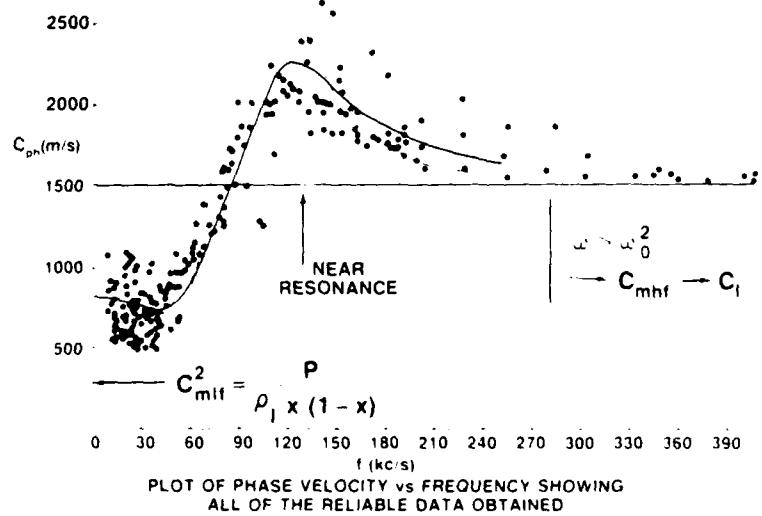
The expression for the sonic speed poses the question of whether the gas compressibility is described by an isothermal or adiabatic process, especially since the single phase sonic speed is known to be adiabatic. However, in the case of an air-bubble/water mixture, the controlling physical factor is the transfer of the heat generated in bubble compression to the surrounding liquid. If the transfer is rapid, then the bubble oscillation is isothermal, ( $\partial V/\partial P = -V/P$ ,  $k_g = \gamma P$ ), as compared to the adiabatic condition ( $\partial V/\partial P = -V/\gamma P$ ,  $k_g = \gamma \gamma P$ ). Thus, in use of the above equations one must use either for the adiabatic or isothermal case,  $C_g = C_g \gamma / \sqrt{\gamma}$ . Isothermal conditions are most likely to prevail for air-bubble/water mixture due to the large thermal capacity of water. Examination of the above expressions shows that as  $X \rightarrow 0$ ,  $C_m^{-2} \rightarrow C_e^{-2}$ , and as  $X \rightarrow 1$ ,  $C_m^{-2} \rightarrow C_g^{-2}$ , as one would expect. The striking characteristic revealed by these equations (shown in this vugraph) is the sharp reduction in the sonic velocity at small volume fractions; i.e.,  $X=0.002 \rightarrow C_m=225 \text{ m/sec}$ . These equations may be approximated for the air/water mixture:

$$C_m^{-2} = \frac{\gamma P}{\rho_e X(1-X)} \xrightarrow{\gamma=1} \frac{P}{\rho_e X(1-X)}$$

$$C_m(X=0.5) = 20 \text{ m/sec.}$$



$$\frac{1}{C_{\text{mhf}}^2} = \frac{(1-x)^2}{C_i^2} + \frac{1}{C_{\text{mhf}}^2 ((1-\omega^2/\omega_0^2) + 2i\delta\omega/\omega_0)}$$



### Experimental Verification of Wood's Result:

Karplus (1958) used an Acoustic tube to determine the standing wave pattern as a function of air volume fraction. Karplus's results are shown in parts a and b of this vu-graph. Close agreement was found between calculated and measured sonic speeds at frequencies between 500 and 1000 Hz using Wood's expression. Similar results have also been observed at low frequencies by Campbell and Pitcher (1955).

Sonic speed changes with volume fraction are also observed at higher frequencies above and below the mean resonant frequency of the micro-bubble distribution of the mixture. Part C of this vu-graph shows the phase velocity measurements of Fox at a constant volume fraction as a function of frequency. One observes the progression from low values at low frequencies, to supersonic values near the resonant frequencies of the individual micro-bubbles to sonic values at frequencies much higher than the resonant frequencies of the individual micro-bubbles (the sonic velocimeter region).

It is important to note that most calculations performed at the higher frequencies have used  $k_m = k_1 + kg$  rather than the Wood approach,  $k_m = (1-x)k_1 + xkg$ . Near the resonant frequencies of the micro-bubbles which form the mixture and at low volume fractions, this difference is unimportant. However, the opposite is true as one approaches the low frequencies of interest to this paper. One can show that the correct frequency dependent behavior for a mixture of micro-bubbles radius ( $k_b$ ) is

$$1/c_{mhf}^2 = (1-x)^2/c_1^2 + 1/[C_{mef}^2((1-\omega^2/\omega_0^2) + 2i\delta)] .$$

In this expression  $hf$  and  $lf$  are the high frequency and low frequency values of the sonic speed.  $C_{mef}$  is determined from Wood's expression;  $C_1$  is the sonic velocity of the liquid;  $\delta$  is the dampening of bubble pulsations; and  $\omega_0$  is the radian resonant frequency of a micro-bubble with radius  $k_b$ , and  $x$  is the volume fraction.

The measurements of Fox et al shown in "C" are compared to his expression (the solid line) assuming  $k_m = k_1 + kg$ . Near the resonance frequency region agreement between measurements and calculation is observed. However, a lack of agreement is observed at the low frequencies and low phase speeds. The exact limiting expression using the Wood approximation is also shown. The lower and higher frequency limits as well as the trends are correctly predicted by the expression given here and Wood's approximation. Care must be used in extrapolation of a variety of results which have been published to this low frequency region.

## BUBBLE CLOUDS AS A SOURCE OF SOUND?

THE DENSITY FIELD CAN BE WRITTEN AS

$$4\pi C_0^2 (\rho - \rho_0) = 4\pi P = \int [\partial q / \partial t] dv / R$$

$$= \frac{1}{R} \frac{\partial}{\partial t} \int q dv$$

BUBBLY MIXTURE: COMPRESSION BECAUSE BUBBLES  
CHANGE VOLUME. LIQUID COMPRESSIBILITY NEGLIGIBLE

$$q = -\rho \frac{D \ln(1-x)}{Dt} = \rho \frac{Dx}{Dt}; \quad x \ll 1$$

$$\text{BY DEFINITION} \quad \Delta P = C_m^2 \Delta(1-x) \quad \rho = -\rho C_m^2 \Delta x$$

$$q \approx \frac{-1}{C_m^2} \frac{\Delta P}{\Delta t} \quad \begin{array}{l} \text{TURBULENT PRESSURE FIELD} \\ \text{PSEUDOSOUND} \sim \rho u^2 \\ L \equiv \text{LENGTH SCALE} \end{array}$$

$$\text{FOR } dv = L^3 \quad L/u \equiv \text{TIMESCALE}$$

$$\begin{aligned} 4\pi C_0^2 (\rho - \rho_0) &= \frac{1}{R} \frac{\Delta \int q dv}{\Delta t} = \frac{1}{R} \frac{qL^2}{L/u} \\ &= \frac{L}{R} \bar{u} \left( \frac{-1}{C_m^2} \right) \rho u^2 / L/u \\ \rho - \rho_0 &= \frac{-L}{4\pi R} \cdot \frac{u^4}{C_0^4} \cdot \frac{C_0^2}{C_m^2} = \frac{-L}{4\pi R} \cdot m^4 \cdot (C_0/C_m)^2 \end{aligned}$$

"A CLOUD OF BUBBLY FLOW RADIATES VERY MUCH  
MORE EFFICIENTLY THAN THE TURBULENCE ALONE.....  
BY  $(C_0/C_m)^4$ .  $(C_0/C_m)^4$  <sub>2%</sub> ~ 1975

### Bubble clouds as a source of sound?

We can examine the radiation of sound from a region of turbulence by following the treatment of sound generation from a turbulence liquid-bubble mixtures in FFWCS- Williams review article "Hydrodynamic Noise".

If the mixture density is given by  $\rho_m = (1-x)\rho_w + \rho_b x$  where  $x$  is the volume fraction, then the equations for conservation of mass and momentum are

$$\text{mass: } \partial \rho_w / \partial t = \partial \rho_w u_i / \partial x_i = \dot{\rho} = -\rho_w \partial \ln(1-x) / \partial t .$$

$$\text{momentum: } \partial (\rho_m u_i) / \partial t + (\partial / \partial x_i) (\rho_m u_i u_j + p_{ij}) = 0 .$$

Take the time derivative of the mass equation and subtract it from the momentum equation to yield:

$$\partial^2 \rho_w / \partial t^2 - c_0^2 \nabla^2 \rho_w = \partial \dot{\rho} / \partial t - \partial f_i / \partial x_i + \partial^2 T_{ij} / \partial x_i \partial x_j ;$$

where

$$T_{ij} = (1-x)\rho_w u_i u_j + p_{ij} - c_0^2 \rho_w \delta_{ij} ; \quad \dot{\rho} = -\rho_w \partial \ln(1-x) / \partial t .$$

These equations describe the density fluctuations and as shown by FFWCS-Williams, when the Mach number is small

$$\partial \dot{\rho} / \partial t > \partial f_i / \partial x_i > \partial^2 T_{ij} / \partial x_i \partial x_j .$$

Consequently the solution for the density field may be written as

$$4\pi(\rho - \rho_0) c_0^2 = 4\pi P = \int [\partial \dot{\rho} / \partial t] dV / R \approx (1/R) \partial \dot{\rho} / \partial t \int \dot{\rho} dV ,$$

where we have assumed  $\dot{\rho}$  is a compact source. We may estimate the sound production by replacing the infinitesimal quantities with approximate values.

$$\dot{\rho} = \rho_w \partial \ln(1-x) / \partial t \approx (\rho / (1-x)) \Delta x / \partial t \stackrel{x \ll 1}{\sim} \rho \Delta x / \partial t .$$

$$\Delta P = C_m^2 \Delta \rho \approx C_m^2 \Delta (1-x) \rho_w = -C_m^2 \rho_w \Delta x_j \quad \Delta t \approx l / u_j \quad \Delta V \approx l^3 ,$$

and within  $\Delta V$  we note  $\Delta P = u_j^2 \rho$ . With these approximations we find

$$\rho - \rho_0 = (1/4\pi R C_m^2) \partial \dot{\rho} / \partial t \int \dot{\rho} dV \rightarrow (-l/4\pi R) \rho \cdot (4/c_0)^4 \cdot C_m^2 / l^3 .$$

$$P = (-\rho e/4\pi R) C_0^2 \cdot (4/C_0)^4 \cdot (C_0/C_m)^2.$$

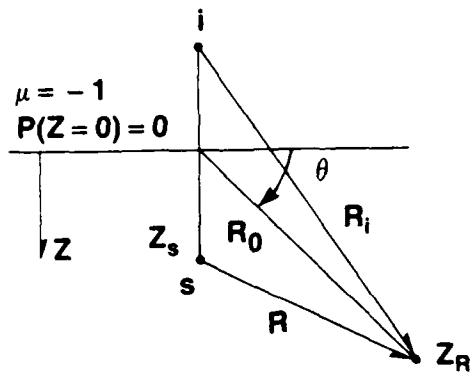
$$I = P^2/\rho c = (-\rho e/4\pi R)^2 C_0^3 \cdot (4/C_0)^8 (C_0/C_m)^4$$

The turbulence will radiate sound with an intensity as shown in the above equation when  $C_m \rightarrow C_0$ . Hence, the ratio of the sound radiated from the bubbly mixtures is seen to be

$$I_m/I_t = (C_0/C_m)^4 \xrightarrow{x=0.2\%} 1,975.$$

Thus, a bubbly mixture will radiate sound 1,975 more intense than the turbulence alone in the absence of the pressure release surface.

## THE SURFACE IMAGE INTERFERENCE EFFECT



$$\sin\theta = Z_R/R_0$$

$$|R_0| \sim |R| \sim |R_i|$$

- THE VELOCITY POTENTIAL AT R,

$$\psi = \frac{v_0}{4\pi |R|} \exp(i\mathbf{k} \cdot \mathbf{R} - i\omega t). \quad 1)$$

- THE RADIATED PRESSURE AT R DUE TO THE POINT SOURCE, S,

$$P_s = \frac{i\omega \rho v_0}{4\pi |R|} \exp(i\mathbf{k} \cdot \mathbf{R} - i\omega t) \xrightarrow{\longrightarrow} \frac{P_0}{|R|} \exp(i\mathbf{k} \cdot \mathbf{R}). \quad 2)$$

- THE PRESSURE FIELD AT R,  $P_T = P_s + P_i$ .

$$P_T/P_0 = \frac{\exp(i\mathbf{k} \cdot \mathbf{R})}{|R|} + \frac{\mu \exp(i\mathbf{k} \cdot \mathbf{R}_i)}{|R_i|} \quad 3)$$

$$\begin{aligned} |P_T/P_0|^2 &= \frac{1}{R_0^2} \left\{ 1 + \mu^2 + 2\mu - 4\mu \sin^2(kZ_s Z_R/R_0) \right\} \\ &= \frac{1}{R_0^2} \left\{ 1 + \mu^2 + 2\mu - 4\mu (16\pi^2) (Z_s/\lambda)^2 (Z_R/R_0)^2 \right\} \stackrel{\mu = -1}{=} \frac{16\pi^2}{R_0^2} \left( \frac{Z_s}{\lambda} \right)^2 \sin^2\theta. \end{aligned} \quad 4)$$

THE SURFACE IMAGE INTERFERENCE EFFECT

When a monochromatic point source is placed a distance  $z_s$  beneath a pressure release surface, an interference occurs which is referred to as the LLOYD's mirror effect. Shown on this vugraph are equations (3,4) for the radiation characteristic of a point source,  $z_s/\lambda \leq 1/4$ , beneath a pressure release surface. When  $\mu = -1$ , the expression for this dipole radiation

$$I/I_o = |P_T/P_o|^2 \approx (16\pi^2/R_o^2) (z_s/2)^2 \sin^2 \theta$$

shows that  $I \rightarrow 0$  as  $z_s \rightarrow 0$ , where  $I_o$  is the point source or monopole intensity.

In our particular problem, each incremental volume of bubbly liquid will have an image in the pressure release surface which results in a dipole-like radiation characteristic. However, regions near the surface radiate at a reduced factor of  $(z_s/\lambda)^2$ .

When a dipole source of sound is placed beneath the surface, rather than a monopole source, a quadrupole results. Furthermore, if the bubbly-region is considered as a complex compact source represented by a superposition of monopole, dipole, quadrupole, and higher order sources, then the proximity of the pressure release surface results in a source represented by a dipole and higher order poles. Consequently for the ambient noise problem, the monopole and its image (the dipole) acts as a source of sound.

**THE RADIATION FROM A FLUCTUATING  
VOLUME V WITH A SONIC SPEED  $C_m$  MAY BE  
DESCRIBED BY**

$$P(R, t) = \rho_0 \dot{V}(t) / 4\pi R \quad \text{WHERE } V = 4/3\pi(r_0 + s)^3 \quad 1)$$

THE DRIVING FORCE CAN BE TURBULENCE  $\frac{1}{2} \rho \bar{u}^2$  AND A SIMPLE HARMONIC OSCILLATION ASSUMED.

$$(\rho_1 \cdot 6/3\pi r_0^3) \ddot{s} + h\dot{s} + Ks = P f_0 4\pi r_0^2, \dot{V}(t) = -4\pi \omega^2 r_0^2 \dot{s} \quad 2)$$

CONSEQUENTLY

$$|P(R, t)| = \frac{3 \rho_1 \omega^2 r_0^3}{(1-x)} \left( \frac{\bar{u}^2}{C_0^2} \right) \left( \frac{C_0}{C_m} \right)^2 \frac{|f(\omega, \omega_0)|}{R}; \quad m = u/c_0 \quad 3)$$

WHERE  $|f(\omega, \omega_0)|$  IS THE RESONANT TRANSFER FUNCTION.

A FURTHER ASSUMPTION OF A SIMPLE IMAGE INTERFERENCE EFFECT YIELDS

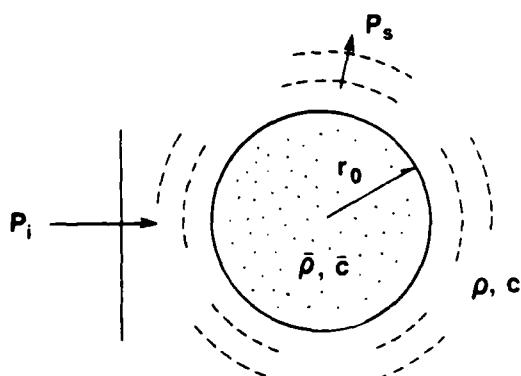
$$|P(R, t)|^2 = \left[ \frac{3 \rho_1 \omega^2 r_0^3 m^2 (C_0/C_m)^2}{(1-x)} \right]^2 \frac{4 |f|^2}{R^2} \left[ \frac{2\pi Z_s}{\lambda} \right]^2 \sin^2 \theta \quad 4)$$

THUS WE HAVE A POSSIBLE SOURCE OF SOUND WHICH HAS A DIPOLE NATURE.  
WHAT ABOUT  $\omega_0$ ?

THE RADIATION FROM A FLUCTUATING VOLUME V.

If we have a region of space with a volume  $V$  that has different sonic properties than its surroundings and if the volume oscillates under the influence of an external force then the radiated pressure from such a region is given by  $P(R,t) = \rho \tilde{V} / 4\pi R$  (equation 1). The driving force could be the turbulence itself or an incident wave, both of which would have a wave length  $\lambda > \lambda_0$ . Two approaches can be used to derive an expression for the radiated pressure field. The first is heuristic and is shown on this vugraph. The driving force is taken to be the turbulence  $\rho_0 = \frac{1}{2} \rho \bar{u}^2$  and a simple harmonic oscillation is assumed. This simple harmonic oscillator equation (2) yields an expression for the radiated pressure with the  $m^2(C_0/c_m)^2$  describing the efficiency of the regions as a radiator and  $|f(w, w_0)|$  describing the resonant harmonic oscillator behavior (equation 3). This monopole when placed under the pressure release surface becomes a dipole with an intensity radiation efficiency on the order of  $m^4(C_0/c_m)^4 [2\pi^2 c / \lambda]^2$  (equation 4). Thus the degree of turbulence, the compressibility of the mixture, and the proximity of the source to the pressure release surface determine the radiation characteristic.

## THE COMPLIANT SPHERE



$$\lambda > > r_0 > > r_b$$

THE INCIDENT PLANEWAVE  
PRESSURE

$$P_i = P_0 \exp(i\omega t - ikr) = P_0 \exp(i\omega t) \sum_{m=0}^{\infty} i^m (2m+1) P(\theta) J_m(k, r) \quad 1)$$

PARTICLE VELOCITY

$$v_i = \left( \frac{-1}{i\omega\rho} \right) \frac{\partial P_i}{\partial r} \quad 2)$$

$$\begin{aligned} \text{BOUNDARY CONDITIONS: } (P_i + P_s)r_0 &= \bar{P}(r_0) \\ (v_{in} + v_{sn})r_0 &= \bar{v}_n(r_0) \end{aligned} \quad 3)$$

WHERE

$$P_s = \exp(i\omega t) \sum_{m=0}^{\infty} a_m P(\theta) G_m(kr) \exp(-i\epsilon_m(kr)) \quad 4)$$

$$\bar{P} = \exp(i\omega t) \sum_{m=0}^{\infty} \bar{a}_m P(\theta) J_m(\bar{k}r) \quad 5)$$

REFERENCE RSCHEVKIN, (350 TO 390)

THE OSCILLATION OF A COMPLIANT SPHERE

The generation and scattering of sound from a compliant sphere immersed in a fluid can be found in classical texts on the theory of acoustics such as Rschevkin (1963) and Morse (1948). The treatment concerning the radiation from compact source regions (Lighthill, 1978) when the characteristic demension,  $1 < \lambda/2\pi$  shows that a compact region can be considered as a superposition of poles, i.e. monopole, dipole, etc. In the case of significant volume fluctuation, the resultant field is described by an equivalent monopole source whose strength is proportional to the volume pulsation. This reasoning was applied in our previous estimation of the source strength to describe the radiation from a bubble plume.

Here we assume that the bubbly region is compact with a generalized radius,  $r_0$  and composed of micro-bubbles with resonant frequencies far above the frequency of excitation. Bouyancy forces and a restoring force such as surface tension are not required. The properties of the bubbly region are described by a mixture speed  $\bar{c}$  and density  $\bar{\rho}$ . The compressibility of the mixture  $\bar{\rho}\bar{c}^2$ . The micro-bubbles supply the compressibility and the liquid the inertia. The source of excitation is assumed to be an incident plane wave expanded in terms of spherical harmonics (eq. 1). However, any forcing field such as the pseudo-sonic field in turbulent flow, which has a wave length greater than the compact region, can be expanded in such a series and yield similar results. The physical reason is that the properties of the bubbly region determine its ability to radiate provided there is a source of excitation. This means that a radiator of sound is also a good scatter of sound.

As shown in equations 2 and 3, continuity of pressure and velocity are required at our generalized radius,  $r_0$ . Furthermore, a radiation condition is required at large distances and the field must remain finite within the bubbly region.

The procedure is to assume the scattered wave or radiated wave is a sum of outward propagating spherical waves described by equation 4. The pressure field inside the volume is expanded in terms of spherical Bessel functions of the first kind. When the region is compact and  $\lambda/2\pi > r_0$ , we can solve for the unknown coefficients  $a_m$  and  $\bar{a}_m$  by substitution into the boundary condition equation 3. Equating each  $m$  term, we find the results shown in equations 6 and 7 to  $\Omega/(r_0^3)$ . The  $\bar{a}_0$  term is seen to represent the resonant volume oscillation of the bubbly region.

Resonance occurs when, as shown in equation 8, the real part of the denominator is equal to zero. The result, equation 9, is interesting as the resonant angular frequency is proportional to the compressibility of the bubbly region, generalized stiffness ( $4\pi\bar{\rho}\bar{c}^2r_0$ ) and the generalized inertia ( $4/3\pi r_0^3\rho$ ). This equation

## THE COMPLIANT SPHERE (CON'T)

FOR THE CASE OF  $kr_0 < < 1$ , WE SOLVE THE BOUNDARY CONDITION EQUATIONS FOR EACH  $m$ th TERM SEPARATELY TO YIELD  $O((kr_0)^3)$

$$a_0 = \frac{iP_0(kr_0)^3/3 (1 - x/\bar{x})}{\left[1 - \frac{x}{\bar{x}} \frac{(kr_0)^2}{3}\right] - i(kr_0) \left[\frac{x}{\bar{x}} \frac{(kr_0)^2}{3}\right]} \quad 6)$$

$$a_1 = \frac{P_0(kr_0)^3 (\bar{\rho} - \rho)}{(2\bar{\rho} + \rho)} \quad 7)$$

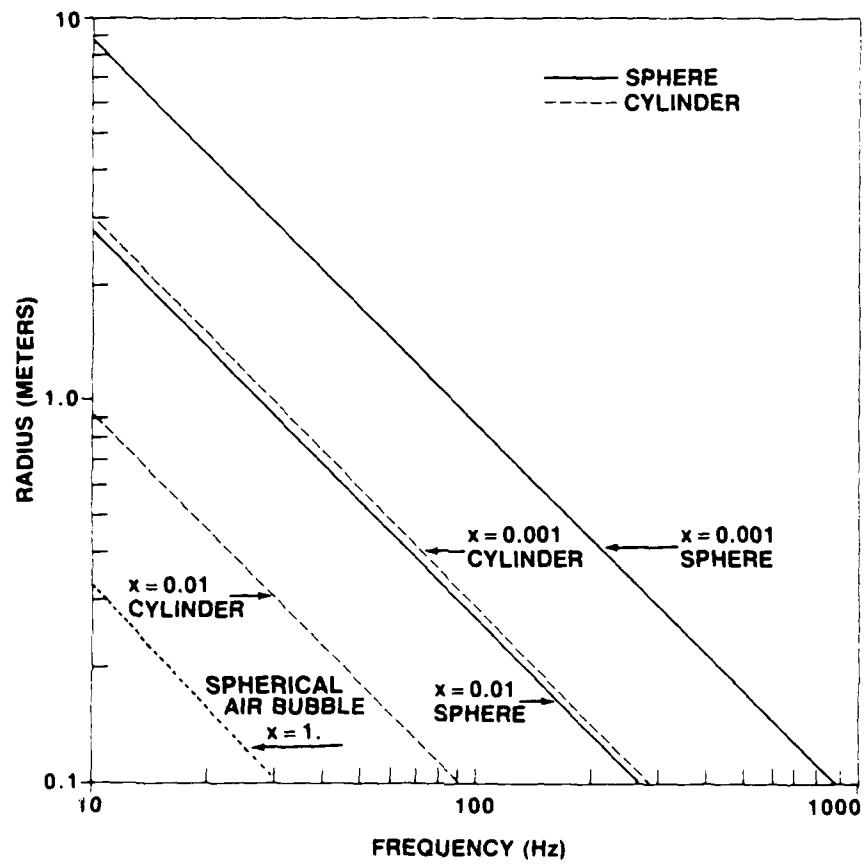
RESONANCE OCCURS WHEN

$$(kr_0)^2 = (2\pi f_0 r_0 / c) = \frac{3\bar{x}}{x} = \frac{3\bar{\rho} \bar{C}^2}{\rho C^2} \quad 8)$$

$$f_0 = \frac{C}{2\pi r_0} \sqrt{\frac{3\bar{\rho} \bar{C}^2}{\rho C^2}} \quad 9)$$

$$f_0 = \frac{\sqrt{3}}{2\pi} \left( \frac{\bar{C}}{r_0} \right) (1 - x)^{1/2} = \frac{0.276}{r_0} \left[ \frac{\gamma P}{x \rho_w} \right]^{1/2} \quad 10)$$

is what one would find for a single isolated gas bubble. In the case of a gas bubble ( $C_g \sim 340 \text{ m/sec}$ ,  $P_0/\rho_c \approx 0.0013$ ) the resonant frequency is  $f_{rb} \approx 680 \text{ Hz} \cdot \text{cm}/\text{ar}_0$ . In the case considered here  $C \sim 200 \text{ m/sec}$ , and we find  $f_{rc} \sim 110 \text{ Hz} \cdot \text{m}/2r_0$ . We note that the resonant cloud of micro-bubble has a resonant frequency ( $f_{rc}/f_{rb} \sim 110/680 = 0.162$ ) of approximately .2 times a gas bubble of the same diameter.



THE SPHERICAL AND CYLINDRICAL BUBBLE CLOUD RESONANT  
FREQUENCY VERSUS RADIUS AND VOID FRACTION

BUBBLE CLOUD RESONANT FREQUENCY

Shown on this vugraph are curves of resonant frequency versus radius of both spherical and cylindrical volumes with bubble void fractions between  $10^{-2}$  and  $10^{-3}$ . These void fractions are consistent with measurements of bubble plumes but must be differentiated from the bubble densities and void fractions within the near surface residual layer.

The curves for the spherical bubble volume are taken from the equations 9 and 10 previously derived in this paper. Also shown are curves for a long cylindrical volume. The estimates for the cylindrical volume are scaled by a comparison of the resonant frequency radius at specific void fractions. This ratio of (Ros/Roc) the spherical radius to cylinder radius is shown to be approximately 3. The results show that the frequency radius dependence of a cylinder with a  $X = .001$  corresponds to a spherical cloud of  $X = .01$ . These curves show that structures between 10 and 50 cm can have resonant frequencies in our range of interest  $\sim 100\text{Hz}$ .

These calculations show that portions of the deep diving bubble plumes have a resonant behavior within the frequency 100 to 200 Hz with reasonable radial sizes.

## SUMMARY

- WHEN BUBBLY MIXTURE VOID FRACTIONS ARE

$$10^{-3} \leq x \leq 10^{-2} (N \approx 10^9/m^3 @ 50\mu m)$$

THEN

$$318 \leq C_m \leq 101 \text{ m/sec}$$

WITH

$$.0063 \leq pC^2/p_w C_w^2 \leq .062$$

UNDER THESE CONDITIONS, COMPOSITE OSCILLATIONS OCCUR

$$p^2 \approx 68 \cdot 10^4 (\mu P_a)^2 [f]^2 \sin^2 \theta$$

WHICH CORRESPOND TO SOURCE LEVELS > 58 dB AND WITH  $[f]^2 = 68 \cdot (.09)$ , ON THE ORDER OF 66 dB re  $\mu P_a$ .

- RESIDUAL BUBBLE LAYERS HAVE DENSITIES AND VOID FRACTIONS LESS THAN THE ABOVE. HOWEVER MEASUREMENTS OF BUBBLE CONCENTRATIONS IN PLUMES AT THE HIGHER SEA STATES ARE VERY UNCERTAIN AND COULD BE ORDERS OF MAGNITUDE GREATER THAN LAYER DENSITIES.
- VOID FRACTION MEASUREMENTS ARE REQUIRED.

SUMMARY

This paper has built on previous results indicating that wind driven noise was important at frequencies near 200 Hz. Prior to wave breaking, this low frequency noise was found consistent with wave-turbulence interaction. After a critical wind speed of 8-10 m/sec (Kerman) bubble plumes composed of micro-bubbles have been observed at depths of tens of meters. The collective oscillations of the micro-bubbles in these plumes driven by the pseudo-sonic and turbulent pressure fields could produce noise which due to the presence of the sea surface would have a dipole character (Carey 1985, 1987).

This paper has presented details of the analysis reported previously. <sup>-2</sup> We have shown that when the void fraction are between  $10^{-2}$  and  $10^{-3}$  that sonic velocity changes are large and can result in sizeable levels of radiated noise as shown in the summary vugraph. The key parameter identified for measurement is the bubble plane void fraction.

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